# Distributed Bayesian Filtering

Each robot can only communicate with its neighboring agents. The set of neighbors of the ith robot is denoted as  and the number of neighbors in  is . The exchanged information is limited to the observation of each robot. Each robot has its individual estimation of the target PDF. Considering the limit of the communication range and bandwidth, no PDF is allowed to be transmitted. The individual PDF of robot i is initialized by the prior function  at time k=0, given all available prior information including past experience and domain knowledge. Once determining the prior distribution, the ith individual PDF at time k, , can be estimated recursively by distributed Bayesian filter based on measurements from the neighborhood of robot i.

## Prediction

Suppose the system is at time step k-1 and the latest update for ith individual PDF is

. The prior PDF is predicted forward to time step k by using the Chapman-Kolmogorov equation:



where  is a probabilistic Markov motion model of target, independent of robot states. This model describes the state transition probability of the target from the prior stateto the destination state . For a static target, 

and the above equation can be reduced to .

## Updating

At time step k, the neighbors of the ith robot, denoted as , the observation of robot i is  and its corresponding observation probability for given target state  , is denoted as  . This is referred to as the observation likelihood for a fixed . It is assumed that all observations are conditionally independent given the current state. Then the target PDF is updated by using the Bayes rule:



 is a normalization factor, given by:

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# Consensus algorithm for measurement fusion

We are interested in distributed computation for the following quantity that depends on the measurements in :

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Let’s define the log-likelihood of the conditional probability.

Then we have:

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The average consensus algorithm to compute  in a distributed way is:



with  that depends on the maximum node degree of the network .