Distributed Search based on Bayesian Filtering and Predictive Control

# Dynamic model of Robot

The dynamic model of robot i is



where  and  is the state of i robot,  can be either homogeneous or heterogeneous.

# Distributed Bayesian Filtering

Each robot can only communicate with its neighboring agents. The set of neighbors of the ith robot is denoted as  and the number of neighbors in  is . The exchanged information is limited to the observation of each robot. Each robot has its individual estimation of the target PDF. Considering the limit of the communication range and bandwidth, no PDF is allowed to be transmitted. The individual PDF of robot i is initialized by the prior function  at time k=0, given all available prior information including past experience and domain knowledge. Once determining the prior distribution, the ith individual PDF at time k, , can be estimated recursively by distributed Bayesian filter based on measurements from the neighborhood of robot i. The upper-script T represents the target, whose position is unknown for robots.

## Prediction

Suppose the system is at time step k-1 and the latest update for ith individual PDF is

. The prior PDF is predicted forward to time step k by using the Chapman-Kolmogorov equation:



where  is a probabilistic Markov motion model of target, independent of robot states. This model describes the state transition probability of the target from the prior stateto the destination state . For a static target,



and the above equation can be reduced to .

## Updating

At time step k, the neighbors of the ith robot, denoted as , the observation of robot i is  and its corresponding observation probability for given target state  , is denoted as  . This is referred to as the observation likelihood for a fixed . It is assumed that all observations are conditionally independent given the current state. Then the target PDF is updated by using the Bayes rule:



 is a normalization factor, given by:

.

## Consensus algorithm for measurement fusion

We are interested in distributed computation for the following quantity that depends on the measurements in :

.

Let’s define the log-likelihood of the conditional probability.

Then we have:

.

The average consensus algorithm to compute  in a distributed way is:



with  that depends on the maximum node degree of the network .

# Finite Horizon Path Planning

The path planning of robots are implemented in a distributed way. The whole robots is planned to converge to a predefined geometry while searching the target. Let simplify the symbol of  to . Note that  in  has totally different meaning from that in , and  denotes the local PDF of target for robot i. The local cost function for the two purposes is designed as follows:



where  is the length of predictive horizon,  is the state of step k in the predictive horizon ,  is the estimated peak of local PDF, calculated by ,  is the desired geometry between i and j, and ,  are given weighting coefficients. The  represents assumed states received from adjacent nodes in , defined as



where  is a virtual value, extended by using the j-th robot model



where  and are the optimal state and input in the predictive horizon at step k-1.

# Appendix

This section gives the consistency proof of the proposed data transmission strategy.

*Proposition* *1.*  Consider a finite set of target position. Using one static binary sensor to detect the single static target, the posterior probability given by the Bayesian estimator will concentrate on the true location of the target, i.e.



where  denotes the true location of the target.

*Proof*: based on the Bayesian estimator,



where the factorization stems from the conditional independence of the observations  .

Since we assume that both the target and the sensor are static,  are i.i.d (independent and identically distributed) samples from the sensor model .

Using the binary sensor model,



where



Take the logarithm of (0.1) and average over sample numbers n:



Utilizing the fact that  are i.i.d and recalling the law of large numbers,



where .

Define



then 

note that the r.h.s obtains the maximum value if and only if  .

Now consider the last term on the r.h.s of (0.2):



Note that:

This is because  is the unique maximum value for .

Then (0.4) converges to  as .

Consider  in (0.2):



Therefore,



This implies that the probability mass will concentrate on the true location of the target.

*Proposition* *2.* Consider a finite set of target position. Using one binary sensor (sensors can move) to detect the single static target, the posterior probability given by the Bayesian estimator will concentrate on the true location of the target, i.e.



where  denotes the true location of the target.

*Proof:*  Based on the Bayes estimator:

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For the purpose of simplicity, we consider a finite set of sensor position . When n tends to infinity, positions in a subset of , defined as , will be visited for infinite times. For any element in , then proposition 1 applies. For elements in the set , their effects on the posterior pdf vanishes as n tends to infinity.

*Proposition* *3.* Using multiple binary sensors to detect the single static target, the posterior probability given by the Bayesian estimator will concentrate on the true location of the target after infinitely many observations, i.e.



where  denotes the true location of the target.

*Proof:* since each sensor’s position is independent from other sensors. Therefore proposition 2 can apply to each sensor. The by similar derivation in the proof of proposition 1, we can prove the proposition 3.