Distributed Search based on Bayesian Filtering and Predictive Control

# Dynamic model of Robot

The dynamic model of robot i is



where and is the state of i robot, can be either homogeneous or heterogeneous.



# Distributed Bayesian Filtering

Each robot can only communicate with its neighboring agents. The set of communication neighbors of the ith robot is denoted as and the number of neighbors in is . Define as the set of the ith robot and its neighbors. The exchanged information is limited to the observation and positions of each robot. Each robot has its individual estimation of the target PDF. Considering the limit of the communication range and bandwidth, no target PDF is allowed to be transmitted. The individual PDF of robot i is initialized by the prior function at time k=0, given all available prior information including past experience and domain knowledge. Once determining the prior distribution, the ith individual PDF at time k, , can be estimated recursively by distributed Bayesian filter based on measurements from the neighborhood of robot i. The superscript T represents the target, whose position is unknown for robots.



## Prediction

Suppose the system is at time step k-1 and the latest update for ith individual PDF is

. The prior PDF is predicted forward to time step k by using the Chapman-Kolmogorov equation:



where is a probabilistic Markov motion model of target, independent of robot states. This model describes the state transition probability of the target from the prior stateto the destination state . For a static target,



and the above equation can be reduced to .



## Updating

At time step k, the neighbors of the ith robot, denoted as , the observation of robot i is and its corresponding observation probability for given target state , is denoted as . This is referred to as the observation likelihood for a fixed . It is assumed that all observations are conditionally independent given the current state of each robot and the target. Then the target PDF is updated by using the Bayes rule:



is a normalization factor, given by:



.



## Distributed Bayesian Filter for Measurement Fusion

### Distributed Measurement Exchange (DME) strategy

We propose a distributed measurement exchange (DME) strategy for the network of robots. Each robot maintains a set of *measurement history* of all agents:



denotes the saved jth robot’s measurement at time . If the ith robot has no information about the jth robot’s measurement, then . Such measurement histories are broadcast among neighbor agents. Upon receiving the data from neighboring agents, the ith agent compares the measurement time labels in with and updates with the newer measurement:



.



The set contains all the indices of observation that are updated. After obtaining measurement at time k and including it into , the ith agent broadcasts to all the neighbor agents and will be utilized by neighbors at time k+1. Figure xxx illustrates the DME process.



It can be proved that in a network of N robots, each robot will obtain the history observations of all other robots within a finite number of communication rounds, as stated in the following propostion:

*Proposition 1* In a network of N robots, any element in will become nonempty when for a connected communication network. Moreover, measurement history of each robot contains history observations of all other robots up to a constant delay, i.e.



where is a constant.



*Proof:*

### Distributed Bayesian Filter for Measurement Fusion

We are interested in distributed computation of the target PDF based on the measurement history. Once updating the history measurement, each robot locally runs the Bayesian filter for updating the target PDF. We first present the distributed Bayesian filter for a static target. Next a distributed Bayesian filter for a moving target will be proposed.

#### Distributed Bayesian Filter for A Static Target

Since the target is static, the prediction step is unnecessary and we remove the subscript of . The update step becomes:





Note that only the observations with indices in are used for updating the target PDF at time k. Because the target is static, observations at different time equally contribute to the estimation process. The effectiveness of this strategy in estimating the target position is proved in the following proposition:



*Proposition 2* (consistency of the DBF) Using multiple binary sensors to detect the single static target, the posterior probability given by the Bayesian estimator will concentrate on the true location of the target after infinitely many observations, i.e.



where denotes the true location of the target. The proof of the proposition is presented in the Appendix.



#### Distributed Bayesian Filter for A Moving Target

This section derives the DBF for a moving target. For the purpose of simplicity, we consider the update of the target PDF of the 1st robot with new measurement .



Following the Bayesian estimation framework:



Different from the DBF for the static target that utilizes the target PDF from previous time for updating, DBF for the moving target requires the ‘time-aligned’ target PDF  and all available measurement after time k-2. Define the set , called the *local measurement history,* as the set that contains the previous measurement (not belong to ) necessary for updating the target PDF. In this three-robot example, . The robot needs to update  and over time and implement the formula. Algorithm 1 gives the general formula of DBF for a moving target. Without loss of generality, assume  and let .



For the ith robot

* Initialize 
* At the time k,
  + Update ‘time-aligned’ target PDF from 



* + Update the target PDF



For the network with N robots, the space complexity is.

# Finite Horizon Path Planning

The path planning of robots are implemented in a distributed way. The whole robots is planned to converge to a predefined geometry while searching the target. Let simplify the symbol of to . Note that in has totally different meaning from that in , and denotes the local PDF of target for robot i. The local cost function for the two purposes is designed as follows:



where is the length of predictive horizon, is the state of step k in the predictive horizon , is the estimated peak of local PDF, calculated by , is the desired geometry between i and j, and , are given weighting coefficients. The represents assumed states received from adjacent nodes in , defined as



where is a virtual value, extended by using the j-th robot model



where and are the optimal state and input in the predictive horizon at step k-1.



# Appendix

This section gives the consistency proof of the proposed data transmission strategy.

*Proposition* *1.*  Consider a finite set of target position. Using one static binary sensor to detect the single static target, the posterior probability given by the Bayesian estimator will concentrate on the true location of the target, i.e.



where denotes the true location of the target.



*Proof*: based on the Bayesian estimator,



where the factorization stems from the conditional independence of the observations .



Since we assume that both the target and the sensor are static, are i.i.d (independent and identically distributed) samples from the sensor model .



Using the binary sensor model,



where



Take the logarithm of (0.1) and average over sample numbers n:



Utilizing the fact that are i.i.d and recalling the law of large numbers,



where .



Define



then



note that the r.h.s obtains the maximum value if and only if .



Now consider the last term on the r.h.s of (0.2):



Note that:



This is because is the unique maximum value for .



Then (0.4) converges to as .



Consider in (0.2):



Therefore,



This implies that the probability mass will concentrate on the true location of the target.

*Proposition* *2.* Consider a finite set of target position. Using one binary sensor (sensors can move) to detect the single static target, the posterior probability given by the Bayesian estimator will concentrate on the true location of the target, i.e.



where denotes the true location of the target.



*Proof:*  Based on the Bayes estimator:

.



For the purpose of simplicity, we consider a finite set of sensor position . When n tends to infinity, positions in a subset of , defined as , will be visited for infinite times. For any element in , then proposition 1 applies. For elements in the set , their effects on the posterior pdf vanishes as n tends to infinity.



*Proposition* *3.* Using multiple binary sensors to detect the single static target, the posterior probability given by the Bayesian estimator will concentrate on the true location of the target after infinitely many observations, i.e.



where denotes the true location of the target.



*Proof:* since each sensor’s position is independent from other sensors. Therefore proposition 2 can apply to each sensor. The by similar derivation in the proof of proposition 1, we can prove the proposition 3.

*Proposition 4.* Using multiple binary sensors to detect the single static target and the DBE measurement fusion method, the posterior probability given by the Bayesian estimator will concentrate on the true location of the target after infinitely many observations, i.e.

where  denotes the true location of the target.